

Ratio of $\bar{\Lambda}/\Lambda$ in Semi-inclusive Electroproduction

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Abstract

It is shown that the $\bar{\Lambda}/\Lambda$ cross section ratio in semi-inclusive electroproduction of Λ and $\bar{\Lambda}$ hyperons in deep inelastic scattering of charged lepton on a nucleon target, can provide useful information on the quark to Λ fragmentation functions. This ratio is calculated explicitly in a quark-diquark model, a pQCD based analysis, and an SU(3) symmetry model, with three different options for the contribution from the unfavored fragmentation functions. The x -dependence of this ratio is sensitive to the ratio of unfavored fragmentation functions over favored fragmentation functions, $D_{\bar{u}}^{\Lambda}(z)/D_u^{\Lambda}(z)$, whereas the z -dependence is sensitive to the flavor structure of the fragmentation functions, i.e., the ratio $D_u^{\Lambda}(z)/D_s^{\Lambda}(z)$. Future measurements by the HERMES Collaboration at DESY can discriminate between various cases.

Key words: Λ hyperon, deep inelastic scattering, semi-inclusive electroproduction, fragmentation function

PACS: 14.20.Jn, 13.60.-r, 13.85.Ni, 13.87.Fh

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The structure of the Λ hyperon is one of the main problems of hadron physics, and is under active investigation both theoretically [1] and experimentally [2,3,4,5,6,7]. Due to its short life time and neutral charge, the Λ cannot be used as target nor as beam, therefore it is difficult to measure the quark distributions of the Λ directly. The most effective means to investigate the structure of the Λ is through the quark to Λ fragmentation in various processes [8,9]. However, there is still no available experimental measurement about the relations between different flavor-dependent quark to Λ fragmentation functions, such as the relation between the favored and unfavored fragmentation functions $D_u^\Lambda(z)$, $D_s^\Lambda(z)$ and $D_{\bar{u}}^\Lambda(z)$, $D_{\bar{s}}^\Lambda(z)$, and the relation between the flavor structure of the favored fragmentation functions $D_u^\Lambda(z)$ and $D_s^\Lambda(z)$.

The purpose of this letter is to show that the $\bar{\Lambda}/\Lambda$ ratio of semi-inclusive electroproduction cross sections in deep inelastic scattering of charged leptons on a nucleon target is a sensitive physical quantity which can provide useful information about the relations between different quark to Λ fragmentation functions. We will calculate the $\bar{\Lambda}/\Lambda$ cross section ratio in three different models: a quark-diquark model [8,10], a perturbative QCD (pQCD) based analysis [8,10], and an SU(3) symmetry model [11], with also three different options for the contribution from the unfavored fragmentation functions [12]. We will show that the x -dependence is sensitive to the ratio of the unfavored fragmentation function $D_{\bar{u}}^\Lambda(z)$ to the favored fragmentation function $D_u^\Lambda(z)$, i.e., $D_{\bar{u}}^\Lambda(z)/D_u^\Lambda(z)$, whereas the z -dependence is sensitive to the flavor structure of the fragmentation functions, i.e., the ratio of $D_u^\Lambda(z)/D_s^\Lambda(z)$. Future measurements by the HERMES Collaboration at the Deutsche Elektronen-Synchrotron (DESY) [13] should be able to discriminate between these various cases.

We parametrize the quark to Λ fragmentation functions $D_q^\Lambda(z)$ by adopting the Gribov-Lipatov relation [14]

$$D_q^\Lambda(z) \propto q^\Lambda(x), \tag{1}$$

in order to connect the fragmentation functions with the quark distribution functions $q^\Lambda(x)$ of the Λ . More explicitly, we adopt a general form to relate

fragmentation and distribution functions, as follows [12]

$$\begin{aligned} D_V^\Lambda(z) &= C_V(z) z^\alpha q_V^\Lambda(z), \\ D_S^\Lambda(z) &= C_S(z) z^\alpha q_S^\Lambda(z), \end{aligned} \tag{2}$$

where a distinction between the valence (V) and the sea (S) quarks is explicit. The above formulae are always correct, since $C_V(z)$ and $C_S(z)$ are in principle arbitrary functions. We should consider Eq. (2) as a phenomenological parametrization for the fragmentation functions of quarks and antiquarks, as follows

$$\begin{aligned} D_q^\Lambda(z) &= D_V^\Lambda(z) + D_S^\Lambda(z), \\ D_{\bar{q}}^\Lambda(z) &= D_S^\Lambda(z). \end{aligned} \tag{3}$$

Three options were found [12] to fit quite well the available experimental data of proton production in e^+e^- inelastic annihilation: (1) $C_V = 1$ and $C_S = 0$ for $\alpha = 0$, (2) $C_V = C_S = 1$ for $\alpha = 0.5$, and (3) $C_V = 1$ and $C_S = 3$ for $\alpha = 1$. We adopt these three options to reflect the relation between unfavored and favored fragmentation functions of the Λ .

There is no direct measurement of the quark distributions of the Λ . But we can relate the quark distributions between the proton and the Λ by assuming SU(3) symmetry between the proton and the Λ [11]

$$\begin{aligned} u_V^\Lambda(x) &= d_V^\Lambda(x) = \frac{1}{6}u_V(x) + \frac{4}{6}d_V(x), \\ s_V^\Lambda(x) &= \frac{2}{3}u_V(x) - \frac{1}{3}d_V(x), \end{aligned} \tag{4}$$

for valence quarks, and

$$\begin{aligned} \bar{u}^\Lambda(x) &= \bar{d}^\Lambda(x) = \frac{1}{2} [\bar{u}(x) + \bar{s}(x)], \\ \bar{s}^\Lambda(x) &= \bar{d}(x), \end{aligned} \tag{5}$$

for sea quarks. We adopt the CTEQ parametrization (CTEQ5 set 1) [15] of the quark distributions $q(x)$ of the nucleon. In this way, we get a complete set

of quark to Λ fragmentation functions, denoted as the SU(3) symmetry model later on.

It is well known that the flavor structure of u and d quark distributions of the proton is different between the quark-diquark model [16,17,18] and pQCD based analysis [19,20]: the quark-diquark model predicts that $d(x)/u(x) \rightarrow 0$ at $x \rightarrow 1$ whereas a pQCD based approach predicts that $d(x)/u(x) \rightarrow 1/5$. A discrimination between the two models requires very high precision measurement of the structure functions at large x and is difficult. On the other hand, it has been also shown [8] that this flavor structure of the quark distributions at large x is even more significant in the case of the Λ , with a large difference between the ratio of $u^\Lambda(x)/s^\Lambda(x)$: the quark-diquark model predicts that $u^\Lambda(x)/s^\Lambda(x) \rightarrow 0$ at $x \rightarrow 1$, whereas the pQCD based approach predicts that $u^\Lambda(x)/s^\Lambda(x) \rightarrow 1/2$. This will produce a large difference in the ratio of fragmentation functions $D_u^\Lambda(z)/D_s^\Lambda(z)$, which might be more easily accessible experimentally via quark to Λ fragmentation.

The valence quark distributions of the Λ in the quark-diquark model and the pQCD based analysis have been explicitly studied [8,9,10] and we adopt the parametrizations in Ref. [10]. To describe the $\bar{\Lambda}$ fragmentation, it is important to take into account the sea contributions in the model construction. In order to use the sea quark distributions from other parametrization while still keep the flavor structure of the valence quarks as predicted in the two models, we re-scale the valence quark distributions by a factor of $u_{V,SU(3)}^\Lambda(x)/u_{V,th}^\Lambda(x)$, where the subscript “SU(3)” denotes the valence quark distributions of the Λ in the SU(3) symmetry model [11] and “th” denotes the corresponding quantities predicted in the quark-diquark model or the pQCD based analysis [10]. This is done in order to normalize the Λ quark distributions to well known proton quark distribution parametrizations. Notice that the valence u -quark distribution then becomes that of the SU(3) model, while the others get a rescaling factor. In this way we can adopt the sea quark distributions from the SU(3) symmetry model as the sea distributions in the quark-diquark model and the pQCD based analysis, to reflect the contribution from the unfavored fragmentation. Thus we get another two sets of quark to Λ fragmentation functions, denoted as the quark-diquark model and the pQCD based analysis later on. This procedure is done with the main motivation of constructing

realistic quark to Λ phenomenological fragmentation functions, which have some features that come from specific theoretical arguments, i.e., the quark-diquark model and the pQCD based analysis.

It can be seen immediately that the ratio of unfavored to favored fragmentation functions $D_{\bar{u}}^\Lambda(z)/D_u^\Lambda(z)$ is the same in the three models:

$$\frac{D_{\bar{u}}^\Lambda(z)}{D_u^\Lambda(z)} = \frac{D_{\bar{u},SU(3)}^\Lambda(z)}{D_{u,SU(3)}^\Lambda(z)}. \quad (6)$$

We plot this ratio in Fig. 1, and find significant differences between the three options for the relation between unfavored and favored fragmentation functions. The contribution from the unfavored fragmentation is important at small z for options 2 and 3. We also plot in Fig. 2 the ratio $D_s^\Lambda(z)/D_s^\Lambda(z)$ in the three models, and find significant differences between the three options for the unfavored fragmentation, but with almost similar values in the three models. In Fig. 3, we plot the ratio of $D_u^\Lambda(z)/D_s^\Lambda(z)$ in the three models, with also the three options for the unfavored fragmentation. We find significant difference of this ratio in the three models: $D_u^\Lambda(z)/D_s^\Lambda(z) \rightarrow 0$ at $x \rightarrow 1$ in the quark-diquark model, whereas $D_u^\Lambda(z)/D_s^\Lambda(z) \rightarrow 1/2$ in the pQCD based analysis and $D_u^\Lambda(z)/D_s^\Lambda(z) \rightarrow 1/4$ in the SU(3) symmetry model. The flavor structure of the Λ differs significantly at large z in the three models. Both the quark-diquark model and the pQCD based analysis break the SU(3) symmetry relation between the proton and the Λ with explicit z -dependence.

The differential cross section of Λ and $\bar{\Lambda}$ production in semi-inclusive deep inelastic scattering of charged leptons on a nucleon target can be expressed as

$$\frac{d^3\sigma^{\Lambda(\bar{\Lambda})}}{dx dy dz} = \frac{4\pi\alpha^2 S}{Q^4} \left(1 + (1-y)^2\right) \sum_{q,\bar{q}} e_q^2 x q(x) D_q^{\Lambda(\bar{\Lambda})}(z), \quad (7)$$

where $x = Q^2/2M_p\nu$ is the Bjorken scaling variable, $z = E_{\Lambda(\bar{\Lambda})}/\nu$ is the energy fraction of the virtual photon energy transferred to the Λ ($\bar{\Lambda}$), $y = \nu/E_e$ is the fraction of the incident lepton's energy E_e transferred to the hadronic system by the virtual photon, $-Q^2 = -Sxy$ is the squared 4-momentum transfer of the virtual photon, and $S = M_p^2 + m_e^2 + 2M_p E_e$ is the squared 4-momentum sum of the incident lepton and target nucleon system. For the HERMES experiment, the incident lepton energy $E_e = 27.6$ GeV, and we

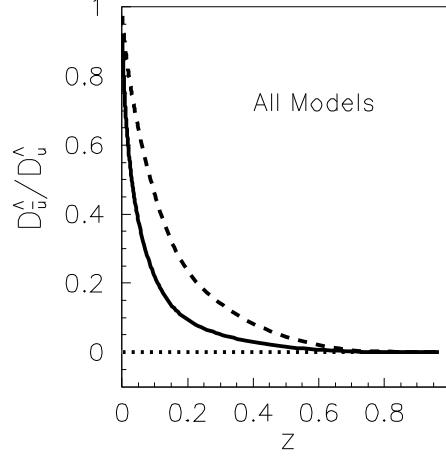


Fig. 1. The ratio $D_u^\Lambda(z)/D_u^\Lambda(z)$ of unfavored and favored fragmentation functions for all three models, with three options of the contribution from unfavored fragmentation: (1) the dotted curve with $C_V = 1$ and $C_S = 0$ for $\alpha = 0$, (2) the solid curve with $C_V = C_S = 1$ for $\alpha = 0.5$, and (3) the dashed curve with $C_V = 1$ and $C_S = 3$ for $\alpha = 1$.

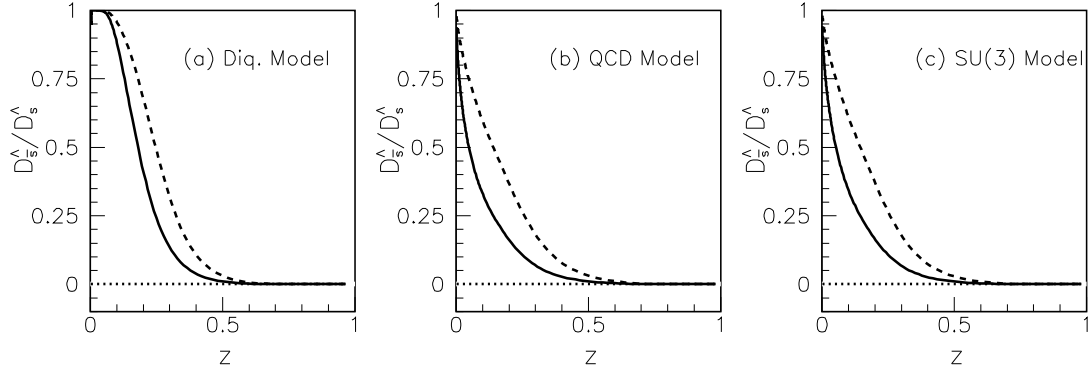


Fig. 2. The ratio $D_s^\Lambda(z)/D_s^\Lambda(s)$ of unfavored and favored fragmentation functions in three different models, with the dotted, solid, and dashed curves corresponding to the three options for the unfavored fragmentation as in Fig. 1.

choose the x range between $0.02 \rightarrow 0.5$ with $\langle x \rangle = 0.09$ for Λ and $\bar{\Lambda}$ events. Therefore we can define the x -dependent ratio of $\bar{\Lambda}/\Lambda$ cross sections

$$R(x) = \frac{\int_0^1 dz \int dy \frac{d^3\sigma^{\bar{\Lambda}}}{dx dy dz}}{\int_0^1 dz \int dy \frac{d^3\sigma^{\Lambda}}{dx dy dz}}, \quad (8)$$

and the z -dependence ratio

$$R(z) = \frac{\int_{0.02}^{0.5} dx \int dy \frac{d^3\sigma^{\bar{\Lambda}}}{dx dy dz}}{\int_{0.02}^{0.5} dx \int dy \frac{d^3\sigma^{\Lambda}}{dx dy dz}}. \quad (9)$$

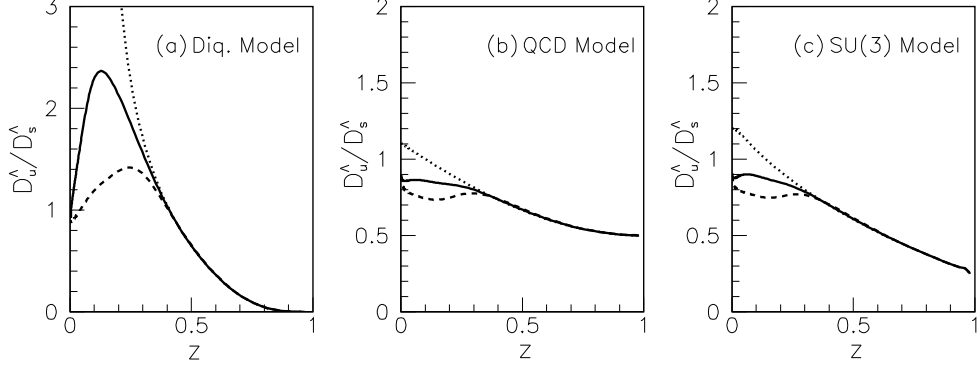


Fig. 3. The ratio $D_u^\Lambda(z)/D_s^\Lambda(z)$ of favored fragmentation functions in three different models, with the dotted, solid, and dashed curves corresponding to the three options for the unavored fragmentation as in Fig. 1.

We calculate these ratios by adopting the CTEQ parametrization of the quark distributions for the proton target and the above three model results for the quark to Λ fragmentation functions.

The x -dependence of the $\bar{\Lambda}/\Lambda$ ratio $R(x)$ is plotted in Fig. 4, from which we find significant differences between the three different options of the unavored fragmentation, but almost no obvious differences between the three models. This can be understood intuitively, as the Λ and $\bar{\Lambda}$ events are dominated by small z contribution in the integration over z , and the ratio $R(x)$ is predominantly determined by the ratio $D_u^\Lambda(z)/D_u^\Lambda(z)$ at small z . Thus the x -dependence of the $\bar{\Lambda}/\Lambda$ ratio is sensitive to the ratio of the unavored fragmentation function $D_u^\Lambda(z)$ over the favored fragmentation function $D_u^\Lambda(z)$. Therefore $R(x)$ is a sensitive physical quantity to provide a discrimination between different options of the contribution from the unavored fragmentation.

The z -dependence of the $\bar{\Lambda}/\Lambda$ ratio $R(z)$ is plotted in Fig. 5, from which we find significant difference between the quark-diquark model and the other two models. We notice that the ratio $R(z)$ increases rapidly as z increases in the quark-diquark model. This can be explained by the fact that the s -quark to Λ (\bar{s} -quark to $\bar{\Lambda}$) fragmentation dominates over the fragmentation of u (\bar{u}) and d (\bar{d}) quarks at large z . The small x region Λ ($\bar{\Lambda}$) events dominate in the integration over x , where the relatively small number of s (\bar{s}) quarks inside the nucleon target cannot be neglected. Thus the ratio $R(z)$ at $z \rightarrow 1$ is predominantly determined by the ratio \bar{s}/s of the proton at small x . However, for the pQCD based analysis and the SU(3) symmetry model, the fragmentation

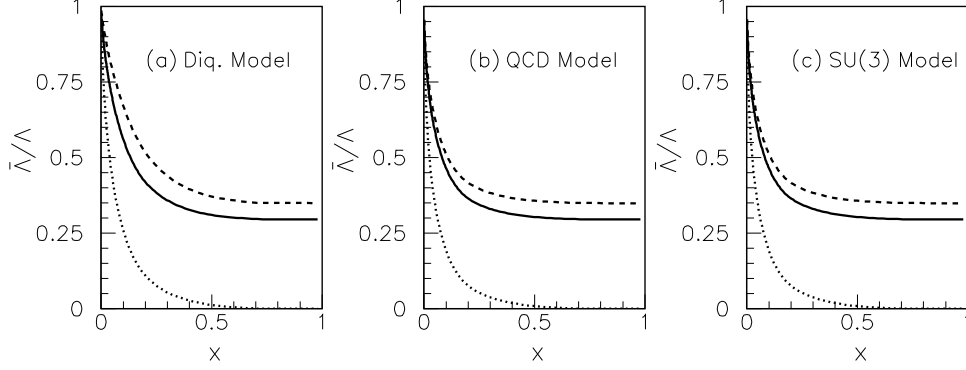


Fig. 4. The x -dependence of the $\bar{\Lambda}/\Lambda$ ratio $R(x)$ in three different models, with the dotted, solid, and dashed curves corresponding to the three options of the unfavored fragmentation as in Fig. 1.

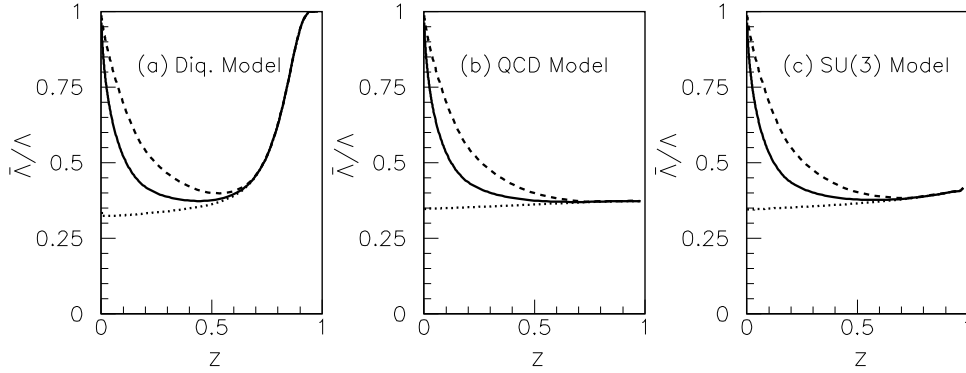


Fig. 5. The z -dependence of the $\bar{\Lambda}/\Lambda$ ratio $R(z)$ in three different models, with the dotted, solid, and dashed curves corresponding to the three options of the unfavored fragmentation as in Fig. 1.

to Λ ($\bar{\Lambda}$) from u (\bar{u}) and d (\bar{d}) quarks are only reduced by a factor of 2 and 4 respectively in relative to the s -quark to Λ (\bar{s} -quark to $\bar{\Lambda}$) fragmentation at large z , thus the ratio of $R(z)$ at $z \rightarrow 1$ is predominantly determined by the ratio \bar{u}/u of the proton at small x . The explicit value of the ratio is sensitive to the x -integrated range, but the qualitative features of the different models remain unchanged, as shown in Fig. 6. Therefore $R(z)$ is a sensitive physical quantity that can discriminate between different flavor structures reflected by the ratio $D_u^\Lambda(z)/D_s^\Lambda(z)$ of favored fragmentation functions.

Although the Gribov-Lipatov relation that we have used in this work is strictly valid only in the limit $z \rightarrow 1$ [14], it provides a reasonable effective guidance for a phenomenological parametrization of the quark to Λ fragmentation functions, and in this sense it is a useful but not essential ingredient of our

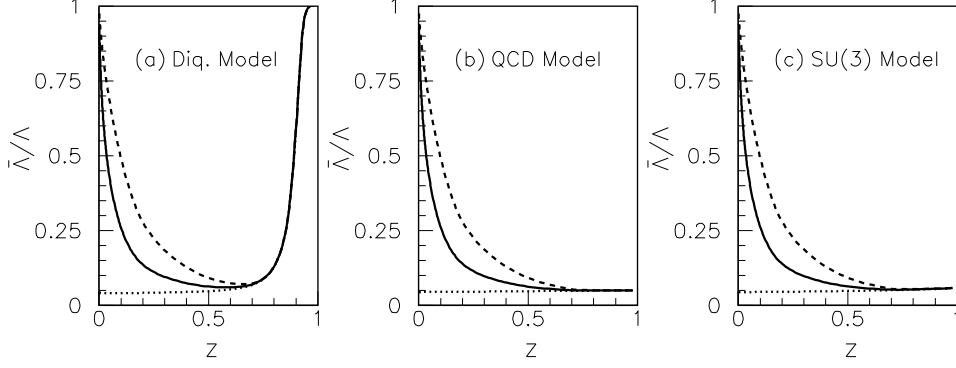


Fig. 6. Same as Fig. 5, but the integrated x range is $[0.2, 0.5]$, rather than $[0.02, 0.5]$ as in Fig. 5.

analysis. Of course, the fact that $R(z)$ gives information about the flavor-dependent fragmentation functions $D_s^\Lambda(z)$ and $D_u^\Lambda(z)$ is independent on the Gribov-Lipatov relation.

In summary, we find in this letter that the $\bar{\Lambda}/\Lambda$ cross section ratio of semi-inclusive Λ and $\bar{\Lambda}$ productions in deep inelastic scattering of charged leptons on a nucleon target is a sensitive physical quantity that can provide useful information on the quark to Λ fragmentation functions. The x -dependence of the ratio can discriminate between different options of the contribution from the unfavored fragmentation, whereas the z -dependence of the ratio can provide a sensitive discrimination between different flavor structure of the favored $D_u^\Lambda(z)$ and $D_s^\Lambda(z)$ fragmentation functions. Thus measurements of the ratio $\bar{\Lambda}/\Lambda$ by the HERMES Collaboration will provide important information on the relations between different flavor-dependent quark to Λ fragmentation functions.

Acknowledgments

This work is partially supported by National Natural Science Foundation of China under Grant Numbers 10025523 and 90103007, by Fondecyt (Chile) grant 1030355, by Alexander von Humboldt-Stiftung (J.-J. Yang), and by Foundation for University Key Teacher by the Ministry of Education (China).

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